BCA-105

B.C.A. (First Year) Examination, 2019

BASIC MATHEMATICS

Paper–V

Time Allowed : Three Hours

Maximum Marks : 100

PART–A   [Marks : 20

Answer all questions (50 words each).

All questions carry equal marks.

PART–B   [Marks : 50

Answer **five** questions (250 words each), selecting **one** question from each Unit. All questions carry equal marks.

PART–C   [Marks : 30

Answer any **two** questions (300 words each).

All questions carry equal marks.

PART-A

1. Answer the following questions :

   (i) Define the Proper subset.

   (ii) Define the Domain and Range of a relation.

   (iii) Find the value of $\lim\limits_{x \to -3} \dfrac{x+3}{x^2+4x+3}$.

   (iv) What is discontinuity ? Define it.

   (v) Find the distance between (0, 0) and (3, 4).

   (vi) Find the equation of a straight line passes through (0, 1) and (2, 3).

   (vii) If $y = x^3 - \dfrac{1}{x^3}$ then find $\dfrac{dy}{dx}$.

   (viii) Write statement of Liebnitz theorem.

   (ix) Evaluate $\int \left(\sqrt{x} + \dfrac{1}{\sqrt{x}} \right)^3 dx$.

   (x) Evaluate $\int \dfrac{1}{1-\sin x} dx$.

PART-B

UNIT-I

2. If A = {1, 2, 3}, B = {2, 3, 4}, C = {1, 3, 4}, D = {2, 4, 5}, then show that

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D).$$

3. A relation R is defined from a set A = {2, 3, 4, 5} to a set B = {3, 6, 7, 10} as follows : $(a, b) \in R \Leftrightarrow a$ is relatively prime to b.

UNIT-II

4. Show that the function $f : R \to R$ given by $f(x) = ax + b$, where $a, b \in R$, $a \neq 0$ is a bijection.

5. A function f(x) is defined as

$$f(x) = \begin{cases} x + 1 & \text{if} \quad -1 < x < 0 \\ x & \text{if} \quad 0 \leq x \leq 1 \\ 2 - x & \text{if} \quad 1 < x \leq 2 \end{cases}$$

Show that it is discontinuous at x = 0 but is continuous at x = 1.

UNIT–III

6. Show that the points A(1, 2), B(2, 5) and C(5, 6) form an isosceles triangle.

7. If $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 2 & 5 \end{bmatrix}$

   then show that $A^3 = A$.

UNIT–IV

8. Differentiate $y = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$ with respect to x.

9. Differentiate $\tan\left\{\log\sqrt{1+x^2}\right\}$ with respect to x.

UNIT–V

10. Evaluate $\displaystyle\int \frac{dx}{\sqrt{4x^2 - 5x + 1}}$.

11. Solve $\displaystyle\int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx$.

PART-C

12. In a class, 21 students play cricket, 26 play hockey, 29 play vollyball. If 14 students play cricket and hockey, 12 play cricket and vollyball, 15 play hockey and vollyball and 8 students play all three games then how many students playing only cricket ?

13. Find the values of a, b and c if

$$\lim_{x \to 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x} = 2.$$

14. Obtain the inverse of the matrix

$$\begin{bmatrix} 2 & 4 & -1 \\ 3 & 1 & 2 \\ 1 & 3 & -3 \end{bmatrix}.$$

   Hence solve the following system of equations
   $2x + 4y - z = 9$, $3x + y + 2z = 7$, $x + 3y - 3z = 4$.

15. Prove that the maximum value of $\left(\dfrac{1}{x}\right)^x$ is $e^{1/e}$.

16. Evaluate $\displaystyle\int_0^{\pi/4} \log(1 + \tan x)\, dx$.